

Adams, William W. & Goldstein, Larry Joel, *Introduction to Number Theory*, Prentice-Hall, 1976, hardcover, xiii + 362 pp., ISBN 0134912829.

After covering standard topics in the first six chapters, the authors devote the remaining five chapters to the Gaussian integers, arithmetic and factorization theory in quadratic fields, applications of the factorization theory to Diophantine equations, and representation of integers as binary quadratic forms.

Adler, Andrew & Coury, John E., *The Theory of Numbers: A Text and Source Book of Problems*, Jones and Bartlett Publishers, 1995, hardcover, xiv + 401 pp., ISBN 0867204729.

A unique feature of this book is that it contains almost 800 solved problems in the main body of the text, *in addition to* its regular problem sets. Besides the standard topics, there are chapters on continued fractions, Pell's equation, and the Gaussian integers (as well as other quadratic extensions).

Anderson, James A. & Bell, James E., *Number Theory with Applications*, Prentice Hall, 1997, hardcover, ix + 566 pp., ISBN 0131901907.

This text requires a bit more sophistication than most introductory texts. In addition to the standard topics, there are chapters on arithmetic functions, continued fractions, Bertrand's postulate, and the relationship between algebra and number theory. There are appendices on logic and proofs, and on Peano's axioms. The applications include random keys, random number generation, two's complement arithmetic, pattern matching, factorization by the Pollard ρ method, unit orthogonal matrices, hashing functions, the Pohlig-Hellman method of computing indices, cryptography, primality testing, and inversions in physics.

Andrews, George E., *Number Theory*, Dover, 1994, paperback, x + 259 pp., ISBN 0131901907. Reprint of 1971 Saunders edition.

This text offers a combinatorial approach to number theory. A number of standard results are proved via combinatorial arguments. There are sections on generating functions, riffing (a type of card shuffling), Tchebychev's theorem, consecutive quadratic residues and nonresidues, and consecutive triples of quadratic residues. Also included are three chapters on partitions and a chapter on geometric number theory. There is much use of infinite series and products and double series, all of which are introduced in appendices. Hints and answers to selected exercises are provided in the back.

Anglin, W. S., *The Queen of Mathematics: An Introduction to Number Theory*, Springer, 1995, x + 389 pp., ISBN 9789401041263 (hardcover) or ISBN 9789401102858 (ebook).

Like Watkins' text, this book follows the order of history, but it is written at a higher level and includes many important results not found in most introductory texts. Continued fractions appear before congruences and quadratic reciprocity. Succeeding chapters deal with equations of the form $x^2 - Ry^2 = C$, classical construction problems, the polygonal number theorem, and analytic number theory. Little advanced mathematics is assumed, but the proofs are terse, making the book most suitable for honors students and first year graduate students. There is much historical information about who discovered what and when.

Apostol, Tom M., *Introduction to Analytic Number Theory*, Springer-Verlag, 1976, hardcover, xii + 338 pp., ISBN 0387901639.

This excellent book presents the standard material (and much more) from the viewpoint of analytic number theory. Discusses arithmetic functions, the distribution of primes numbers, primes in arithmetic progressions, Dirichlet series and Euler products, zeta functions, the prime number theorem, and partitions. There is a sequel, *Modular Functions and Dirichlet Series in Number Theory* (Second Edition), Springer-Verlag, 1990, x + 204 pp.

Baker, Alan, *A Concise Introduction to the Theory of Numbers*, Cambridge University Press, paperback, xiii + 95 pp., 1985, ISBN 0521286549.

A concise yet broad introduction to number theory, with detailed suggestions for further reading. The chapter headings give an idea of its scope: divisibility, arithmetical functions (with sections on average orders and the Riemann zeta function), congruences (including theorems of Fermat, Euler, Wilson, and Lagrange, plus primitive roots and indices), quadratic residues (including quadratic reciprocity and Jacobi symbols), quadratic forms (including representation of integers as sums of two and four squares), Diophantine approximation (including Dirichlet's theorem, continued fractions, transcendental numbers, and theorems of Liouville and Minkowski), quadratic fields, and Diophantine equations (including equations of Pell, Thue, Mordell, Fermat, and Catalan). Proofs are sketchy. No exercises.

Baker, Alan, *A Comprehensive Course in Number Theory*, Cambridge University Press, 2012, xv + 251 pp., hardback, ISBN 9781107019010, paperback ISBN 9781107603790.

This is a sequel to Baker's earlier book, *A Concise Introduction to the Theory of Numbers*. It covers elements of cryptography, factorization, and primality tests; discusses number fields; and exposes readers to

various aspects of analytic number theory including Riemann's zeta function, the prime number theorem, primes in arithmetic progression, and sieve methods; and concludes with a chapter on elliptic curves.

Beiler, Albert H., *Recreations in the Theory of Numbers*, Dover, 1964, paperback, xvi + 349 pp., ISBN 0486210960.

Not a textbook. Full of facts and tables. Fun to read. Topics discussed are not evident from chapter titles, but include perfect, multiply perfect, amicable, and Mersenne numbers, congruences, Fermat's and Wilson's theorems, casting out 9s, base ten number patterns, other bases, periods of decimal fractions, repunit numbers, Euler's phi function, the theory of indices, Pythagorean triangles, sums of squares, Farey series, Fermat numbers and constructible polygons, polygonal numbers, quadratic reciprocity, prime numbers, factorization, Pell's equation, binary quadratic forms, and Fermat's last theorem.

Benjamin, Arthur T. & Brown, Ezra (editors), *Biscuits of Number Theory*, Mathematical Association of America, 2009, xiii + 309 pp., ISBN 9780883853405.

This is a collection of exceptionally well written articles that can be appreciated by anyone who has taken (or is taking) a first course in number theory. Its seven parts are entitled Arithmetic; Primes; Irrationality and Continued Fractions; Sums of Squares and Polygonal Numbers; Fibonacci Numbers; Number-Theoretic Functions; and Elliptic Curves, Cubes and Fermat's Last Theorem. There is a lot here to enjoy.

Bressoud, David M., *Factorization and Primality Testing*, Springer-Verlag, 1989, hardcover, xiii + 237 pp., ISBN 0387970401.

This text focuses on the problem of factoring a large number or proving that it is prime. It is shorter and lower in level than Riesel's text, but does discuss elliptic curves. Emphasis is on the theory behind the algorithms: how they arise and why they work. Algorithms are presented in a sort of "shorthand Pascal". To obtain high precision, the author suggests use of REXX, a language developed by IBM.

Bressoud, David & Wagon, Stan, *A Course in Computational Number Theory*, Key College Publishing, 2000, hardcover, xii + 367 pp., ISBN 1930190107. Includes CD-ROM with software package whose use requires *Mathematica*.

This book is designed to be a one-semester computer-based introduction to number theory. It is a successor, not a sequel, to Bressoud's *Factorization and Primality Testing*. It reflects a decade of advances in factorization and primality testing. The topic of elliptic curves has been omitted for lack of space to do it justice. The detailed discussion of the quadratic sieve has been replaced by a detailed discussion of the continued fraction algorithm. Infrequent programs in pseudocode have been replaced by a rich assortment of *Mathematica* programs on CD-ROM. The reader need not know how to program to use this book. Includes appendix on *Mathematica*.

Burn, R. P., *A Pathway Into Number Theory* (Second Edition), Cambridge University Press, 1997, paperback, xv + 262 pp., ISBN 0521575400.

This book leads the reader through an undergraduate course in number theory by means of a sequence of problems and notes. Answers are provided. It includes chapters on partitions, quadratic forms, the geometry of numbers, continued fractions, and approximation of irrationals by rationals. (See also Roberts' *Elementary Number Theory: A Problem-Oriented Approach*.)

Burr, Stefan A. (editor), *The Unreasonable Effectiveness of Number Theory: AMS Short Course Aug. 6–7, 1991 in Orono, Maine*, AMS Proc. Symp. Applied Math. **46** 1992, hardcover, xii + 125pp., ISBN 0821855018

This volume is devoted to applications of number theory. It consists of six lectures: "The Unreasonable Effectiveness of Number Theory in Physics, Communication and Music" by Manfred R. Schroeder, "The Reasonable and Unreasonable Effectiveness of Number Theory in Statistical Mechanics" by George E. Andrews, "Number Theory and Dynamical Systems" by J. C. Lagarias, "The Mathematics of Random Number Generators" by George Marsaglia, "Cyclotomy and Cyclic Codes" by Vera Pless, and "Number Theory in Computer Graphics" by M. Douglas McIlroy.

Burton, David M., *Elementary Number Theory* (Seventh Edition), McGraw-Hill, 2011, hardcover, xii + 436 pp., ISBN 9780073383149.

A fine introductory text. Somewhat formal and proof-oriented. Historical comments included. Excellent problems. Chapters on cryptography, Fibonacci numbers, continued fractions, and modern developments.

Campbell, Duff, *An Open Door to Number Theory*, MAA Press (an imprint of the AMS), 2018, hardcover, xii + 283 pp., ISBN 9781470443481.

This textbook is designed for students who have had exposure to proof writing, but not necessarily to abstract algebra. It will prepare them for a follow-up course on algebraic number theory. The order of topics is nonstandard and there is an emphasis on calculation. Continued fractions are covered early in the book, as are polynomial congruences. Quadratic extensions of the integers are emphasized. There are nine projects on topics not usually covered in a first course on number theory, including Bernoulli numbers and polynomials, polygons in lattices, the p -adic numbers, quadratic extensions of the integers, arithmetic functions and Dirichlet series, and the geometry of continued fraction convergents. The author tries, especially in the projects, to show how number theorists use tools from other areas of mathematics.

Chowdhury, K. C., *A First Course in Number Theory*, Asian Books Private Limited, 2004, x + 30 pp., ISBN 9788186299654.

This introductory text starts with the construction of the natural numbers from Peano's axioms and then covers the standard topics from elementary number theory, but also includes the theory of indices; a full chapter on arithmetic functions; a chapter on Farey sequences, continued fractions, and Pell's equation; and a final chapter containing 79 number theoretic problems related to Mathematics Olympiads along with their solutions.

Cohn, Harvey, *Advanced Number Theory*, Dover, 1980, paperback, xi + 276 pp., ISBN 048664023X. First published by John Wiley & Sons, Inc. in 1962 under the title *A Second Course in Number Theory*.

This book concentrates on algebraic number theory. It is divided into three parts: I. Background Material (including characters, quadratic integers, basis theorems and their applications), II. Ideal Theory in Quadratic Fields (unique factorization and units, unique factorization into ideals, norms and ideal classes, and class structure in quadratic fields), and III. Applications of Ideal Theory (class number formulas and primes in an arithmetic progression, quadratic reciprocity, quadratic forms and ideals, and compositions, orders, and genera), plus a Concluding Survey (cyclotomic fields and Gaussian sums, class fields, and global and local viewpoints). More a monograph than a text. Relatively few exercises.

Conway, John H. & Guy, Richard K., *The Book of Numbers*, Springer Verlag, 1996, hardcover, ix + 310 pp., ISBN 038797993X.

This is not a textbook on number theory, but rather a wonderful book about all kinds of numbers, their history, their properties, and how they relate to each other and to the real world. Many topics that we will discuss appear here, such as triangular numbers, Mersenne numbers, perfect numbers, Fermat numbers, decimal fractions, Euler's totient function, Wilson's theorem, and numbers in other bases, together with many other topics that will surprise and delight you.

Coppel, W. A., *Number Theory: An Introduction to Mathematics* (Second Edition), Springer, 2009, xiii + 610 pp., hardcover ISBN 9780387894850, ebook ISBN 9780387894867.

This comprehensive text emphasizes theoretical mathematics over computational mathematics. It is divided into two parts. Part A deals with elementary number theory, including continued fractions, Hadamard's determinant problem (not found in other texts), and Hensel's p -adic numbers. Part B, which is more advanced, is intended to provide an undergraduate with some idea of the scope of mathematics today. It consists of chapters on the arithmetic of quadratic forms, the geometry of numbers, the number of primes, characters of finite groups, uniform distribution and ergodic theory, elliptic functions, and connections with number theory. That last chapter includes representations of a natural number as a sum of squares, partitions, cubic curves, further results and conjectures on elliptic curves, and some applications of elliptic curves. An extensive list of references is given at the end of each chapter.

Davenport, Harold, *The Higher Arithmetic* (Eighth Edition), Cambridge University Press, 2008, paperback, 248 pp., ISBN 9780521722360.

Not a text. These lectures give an efficient and well-rounded introduction to number theory. Chapters on continued fractions, quadratic forms, and computers and the theory of numbers.

Dickson, Leonard Eugene, *History of the Theory of Numbers* (3 volumes), Dover. First published in 1919, 1920, and 1923. Volume I, *Divisibility and Primality*, paperback, 512 pp., ISBN 0486442322. Volume II, *Diophantine Analysis*, paperback, 832 pp., ISBN 0486442330. Volume III, *Quadratic and Higher Forms*, paperback, 320 pp., ISBN 0486442349.

This is THE source book in number theory. To find out what was known about any subject in number theory (except quadratic reciprocity) in 1920, together with information on who discovered it and when, look here.

Dudley, Underwood, *Elementary Number Theory* (Second Edition), W. H. Freeman and Co., 1978, hardcover, ix + 249 pp., ISBN 071670076X. Reissued as Dover paperback, 272 pp., ISBN 048646931X.

Informal, readable, well-written elementary text. Large collection of exercises, including a whole section of supplementary problems and an appendix consisting of computer problems. Sections on formulas for primes and bounds for $\pi(x)$ (the number of primes $\leq x$).

Edgar, Hugh M., *A First Course in Number Theory*, Wadsworth, 1988, hardcover, xiii + 139 pp., ISBN 0534085148.

This slim volume—104 pages excluding the appendices, hints/solutions, bibliography, and index—attempts to keep to a clear presentation of the core topics. It is unique in its early introduction and subsequent reliance on p -adic valuations. The chapter headings are unusual: Divisibility Without Primes, Divisibility With Primes, Numerical Congruences, Polynomial Congruences in One Variable, and Carl Friedrich Gauss' Law of Quadratic Reciprocity.

Erdős, Paul & Surányi, János, *Topics in the Theory of Numbers* (Second Edition), Springer, 2003, hardcover, xviii + 287 pp., ISBN 0387953205.

This book is described as a guided tour through number theory. Instead of providing a systematic and exhaustive treatment of the subject, the authors have chosen instead to illustrate the many varied subjects by associating recent discoveries, interesting methods, and unsolved problems. In particular, they discuss combinatorial problems in number theory, a branch of mathematics cofounded and popularized by Paul Erdős. There is a wealth of material in this book, much of it in the exercises, and the authors make a point of telling the reader what is still *not* known about the topics covered.

Erickson, Martin & Vazzana, Anthony, *Introduction to Number Theory*, Chapman & Hall/CRC, 2008, hardcover, 521 pp., ISBN 9781584889373. Second edition, 2016, hardcover, xii + 414 pp., ISBN 9781498717496.

This is a text with both depth and breadth. It covers the usual elementary number theory topics in the first nine chapters, plus material on the construction of the regular 17-gon, cryptographic methods, Hadamard matrices, partitions of integers, continued fractions, and various Diophantine equations. The remaining three chapters discuss analytic number theory (such as Chebyshev's theorem, Bertrand's postulate, the prime number theorem, the Riemann hypothesis, and Dirichlet's theorem), elliptic curves, and applications of logic to number theory, including the negative solution of Hilbert's Tenth Problem. The book features Mathematica and Maple algorithms for many of the important number theoretic calculations, as well as historical notes. No answers to exercises are provided. There is further supplemental material at www2.truman.edu/~erickson/introduction_to_number_theory/.

Erickson, Martin, Vazzana, Anthony, & Garth, David, *Introduction to Number Theory*, (Second Edition), Chapman & Hall/CRC, 2016, hardcover, xii + 414 pp., ISBN 9781498717496.

This edition changes the order of topics and relegates the chapter on logic and number theory to the website tvazzana.sites.truman.edu/introduction-to-number-theory/, along with the examples of *Mathematica* and *Maple* code for doing large computations. For this edition, examples of code in *Sage* are provided on the website too.

Everest, Graham & Ward, Thomas, *An Introduction to Number Theory*, Graduate Texts in Mathematics 232, Springer, 2005, hardcover, ix + 294 pp., ISBN 1852339179.

Graduate level text that uses the Fundamental Theorem of Arithmetic to illuminate much of elementary number theory and illustrates different approaches to number theory by treating topics from analytic, algebraic/geometric, and computational number theory, all in one book. Readers are assumed to have had courses in abstract algebra and analysis, including complex analysis. Chapter headings: A Brief History of Prime, Diophantine Equations, Quadratic Diophantine Equations, Recovering the Fundamental Theorem of Arithmetic, Elliptic Curves, Elliptic Functions, Heights, The Riemann Zeta Function, The Functional Equation of the Riemann Zeta Function, Primes in an Arithmetic Progression, Converging Streams, and Computational Number Theory. The Converging Streams chapter brings together the text's elliptic, analytic, and algebraic threads to describe the class number formula for quadratic fields and the conjectures of Birch and Swinnerton-Dyer.

Flath, Daniel E., *Introduction to Number Theory*, Wiley, 1989, hardcover, xii + 212 pp., ISBN 047160836X.

A text designed to teach (some of) Gauss' *Disquisitiones Arithmeticae*. Covers linear Diophantine equations in several variables, Chebyshev's theorem on the distribution of primes, representation of a positive integer as a sum of k squares ($k = 2, 3, 4$), Gaussian integers, Farey sequences, Minkowski's theorem, quadratic reciprocity, Pell's equation, continued fractions, and binary quadratic forms.

Forman, Sylvia & Rash, Agnes M., *The Whole Truth About Whole Numbers: An Elementary Introduction to Number Theory*, Springer, 2015, xiii + 282 pp., hardcover ISBN 9783319110349, ebook ISBN 9783319110356.

This text is aimed at nonmath and nonscience majors. It includes an introduction to logic and proofs at a level suitable for liberal arts majors, as well as major concepts in number theory accessible to students with two years of high school algebra. The chapter titles are Introduction; Conjectures, Proofs, and Counterexamples; Pythagorean Triples; Prime Numbers; The Euclidean Algorithm; Congruences; Numerical Functions and Special Congruences; and Cryptography. Answers to the odd-numbered exercises are included.

Fraenkel, Abraham A., *Integers and Theory of Numbers*, Dover, 2004, hardcover, 102 pp., ISBN 0486495884. Unabridged and unaltered republication of Scripta Mathematica Studies No. 5, published in 1955 by Yeshiva University, New York.

This little book is concerned with the concept of number and its foundations as well as with selected topics from the theory of numbers. Its chapter headings are: Natural Numbers as Cardinals, Natural Numbers as Ordinals, Theory of Numbers, and Rational Numbers. Within the chapter on the theory of numbers, there are sections on primes numbers and their distribution, partition of the circle, Fermat's simple and last theorems and the concept of congruence, perfect Numbers and amicable Numbers, and algebraic and ideal numbers.

Friedberg, Richard, *An Adventurer's Guide to Number Theory*, Dover, 1994, paperback, 228 pp., ISBN 0486281337.

This is not a text, but an engaging, historically oriented introduction to number theory with no prerequisites other than an understanding of arithmetic and beginning algebra. Still, it explores the subject in surprising depth, explaining methods of Diophantus and moving on to results of Fermat, Euler, Lagrange, and Gauss, including quadratic reciprocity and representation of integers by quadratic forms.

Gauss, Carl F., *Disquisitiones Arithmeticae* (reissue edition), Springer Verlag, 1986, hardcover, xx + 472 pp., ISBN 0387962549.

This is a translation of Gauss' landmark treatise, first published in 1801. One of the greatest mathematicians of all time, Gauss almost singlehandedly turned number theory into a unified, well-developed subject. This book is surprisingly readable, but after the first 100 pages or so it discusses topics beyond the scope of an introductory course.

Gioia, Anthony A., *The Theory of Numbers*, Dover, 2001, paperback, xii + 207 pp., ISBN 0486414493. First published by Markham Publishing Co. in 1970 under the name *The Theory of Numbers: An Introduction*, but the Dover edition adds solutions to selected exercises.

Concise introductory text with material on algebraic and analytic number theory, as well as the geometry of numbers. Discusses the algebraic structure of arithmetic functions under Dirichlet convolution, orders of magnitude of arithmetic functions, and the Erdős-Selberg proof of the prime number theorem. Uses Gaussian integers (to study representations of integers as sums of two squares) and Jacobian integers (to prove the insolvability of Fermat's cubic).

Granville, Andrew, *Number Theory Revealed: An Introduction*, American Mathematical Society, 2019, xviii + 264 pp., ISBN 9781470441579.

This text takes a fresh look at congruences, power residues, quadratic residues, primes, and Diophantine equations and presents hot topics like cryptography, factoring, and primality testing, as well as less often covered topics like the structure of Pascal's triangle mod p , the values represented by binary quadratic forms, and large solutions of equations. Appendices at the ends of the chapters provide additional reading, projects, and references. This book is a shortened version of Granville's more comprehensive book *Number Theory Revealed: A Masterclass*.

Granville, Andrew, *Number Theory Revealed: A Masterclass*, American Mathematical Society, 2019, xxviii + 587 pp., ISBN 9781470441586.

This version of *Number Theory Revealed* is a more comprehensive approach to the core topics, supplemented by additional material in further chapters and appendices. This book is the most up to date introductory treatment of number theory currently available. It even discusses open questions and progress to date on those questions. Granville plans to write three follow-up books on number theory: *The distribution of primes: Analytic number theory revealed*, *Rational points on curves: Arithmetic geometry revealed*, and *Gauss's Disquisitiones Arithmeticae revealed*.

Grosswald, Emil, *Topics from the Theory of Numbers* (Second Edition), Birkhäuser, 1984, hardcover, viii + 333 pp., ISBN 0817630449 or 3764330449.

Contains in-depth treatments of several advanced topics not usually found in introductory texts. The advanced topics include Riemann's zeta function, the prime number theorem, algebraic number fields, the theory of partitions, primes in arithmetic progressions, Diophantine equations (including recent progress), and Fermat's equation.

Guy, Richard K., *Unsolved Problems in Number Theory* (Third Edition), Springer-Verlag, 2004, hardcover, xviii + 437 pp., ISBN 9780387208602.

This book will tell you about problems that have NOT been solved, and gives lots of references to help you find out what has been done to try to solve them.

Hardy, G. H. (Godfrey Harold) & Wright & E. M. (Edward Maitland) (authors), Heath-Brown D. R., & Silverman, Joseph H. (editors) *An Introduction to the Theory of Numbers* (Sixth Edition), Oxford University Press, 2008, 621 pp., hardcover ISBN 9780199219858, paperback ISBN 9780199219865.

This classic text is an advanced treatise from which many other texts draw their material. Chapters on Farey series, representation of numbers by decimals, continued fractions, approximation of irrationals by rationals, quadratic fields, generating functions and order of magnitude of arithmetic functions, partitions, representation as sums of two or four squares, representations by cubes and higher powers, the distribution of primes, Kronecker's theorem, the geometry of numbers, and elliptic curves. The editors have added the chapter on elliptic curves to this edition and have greatly expanded the notes at the ends of the chapters.

Holt, Jeff & Jones, John, *Discovering Number theory*, Freeman, 2001, paperback, xix + 649 pp., ISBN 0716742845.

This large-format paperback asks students to discover number theoretic results with the help of Maple, Mathematica, or a Java-based Web browser. The book is organized into labs, each requiring students to produce a lab report. The chapter summaries are a key part of the text, but they are omitted from the student version so as not to give away the results in advance. The instructor distributes these missing summaries after lab reports have been turned in. Despite the lab format, this text's exercises require lots of proofs as well as calculations.

Hua, Loo-Keng, *Introduction to Number Theory* (translated from the Chinese by Peter Shui), Springer-Verlag, 1982, hardcover, xviii + 572 pp., ISBN 0387108181.

A deep and comprehensive introduction to number theory by a Chinese master of the subject. Almost 600 pages long, this book covers many topics not easily found elsewhere and includes much advanced mathematics. Not easy reading, but fascinating to browse through. More a treatise than a text; very few exercises. Some of the chapter headings are The Distribution of Prime Numbers, Trigonometric Sums and Characters, On Several Arithmetic Problems Associated with the Elliptic Modular Function, The Prime Number Theorem, Continued Fractions and Approximation Methods, Binary Quadratic Forms, Unimodular Transformations, Integer Matrices and Their Applications, p -adic Numbers, Introduction to Algebraic Number Theory, Algebraic Numbers and Transcendental Numbers, and The Geometry of Numbers.

Ireland, Kenneth & Rosen, Michael, *A Classical Introduction to Modern Number Theory* (Second Edition), Springer-Verlag, 1995, hardcover, vii + 389 pp., ISBN 038797329X.

This text focuses on topics that point in the direction of algebraic number theory and arithmetic algebraic geometry. It is aimed at graduate students and upper level undergraduates who have taken undergraduate abstract algebra. Finishes quadratic reciprocity in 65 pages, then goes on to cover cubic, biquadratic, and Eisenstein reciprocity. Discusses Diophantine equations over finite fields and over the rational numbers, as well as zeta functions and elliptic curves.

Jones, Gareth A. & Jones, J. Mary, *Elementary Number Theory*, Springer-Verlag, 1998, paperback, xiv + 301 pp., ISBN 3540761977.

Clear, concise introduction to number theory. Similar in style to a good set of course notes. Goes a little further into several topics than most introductory texts do, assuming some knowledge of abstract algebra. The extras include the generalized Chinese Remainder Theorem, the structure of the group of units in \mathbf{Z}_n , the Dirichlet product of arithmetic functions, the Riemann zeta function, and Minkowski's theorem. Includes hints or solutions for all the exercises.

Khinchin, A. Y., *Continued Fractions*, Dover, 1964, paperback, 106 pp., ISBN 0486696308.

This little book concentrates on just one topic (continued fractions), covering it in detail.

Khinchin, A. Y., *Three Pearls of Number Theory*, Dover, 1998 republication of the English translation of the second revised Russian edition of 1948, originally published by Graylock Press in 1952, paperback, 64 pp., ISBN 0486400263.

This book focuses on three important advanced topics in the theory of numbers: Van der Waerden's theorem on arithmetic progressions, the Landau-Schnirelman hypothesis and Mann's theorem, and an elementary solution to Waring's problem.

Kirch, Allan M., *Elementary Number Theory*, Intext, 1974, hardcover, xi + 339 pp., ISBN 0700224564.

An introduction to number theory with a strong emphasis on computer programming in FORTRAN. Numerous programs and programming exercises (most with solutions) are provided.

Kirtland, Joseph, *Identification Numbers and Check Digit Schemes*, Mathematical Association of America, 2001, paperback, 174 pp., ISBN 0883857200.

Check digit schemes are an application of number theory that we use all the time without realizing it. This book explores the subject in a readable and elementary manner, basing much of its exposition on accessible and interesting articles by Joseph A. Gallian.

Koblitz, Neal, *A Course in Number Theory and Cryptography* (Second Edition), Springer-Verlag, 1995, hardcover, 235 pp., ISBN 0387942939.

For advanced students. Discusses cryptography, primality, and factoring, including the elliptic curve method.

Koshy, Thomas, *Elementary Number Theory with Applications* (Second Edition), Academic Press, 2007, hardcover, xi + 771 pp., ISBN 0123724872.

Long introductory book with detailed examples and proofs. Sections on polygonal and pyramidal numbers, number patterns, the Euclidean algorithm and matrices, linear Diophantine equations and matrices, modular designs, the p -queens puzzle, round-robin tournaments, and the perpetual calendar. Chapter on cryptology. The second edition added a chapter on continued fractions.

Koshy, Thomas, *Fibonacci and Lucas Numbers with Applications, Volume 1* (Second Edition), Wiley, 2018, xx + 680 pp., hardcover, ISBN 9781118742129.

Extremely detailed treatment of Fibonacci and Lucas numbers, their properties, generalizations, occurrences in nature, and connections with other parts of mathematics. This is the first of two volumes, which together expand upon Koshy's original 2001 volume (with the same title) on this subject.

Koshy, Thomas, *Fibonacci and Lucas Numbers with Applications, Volume 2*, Wiley, 2019, xii + 729 pp., hardcover, ISBN 9781118742082.

By replacing the recurrence relation $f_n = f_{n-1} + f_{n-2}$ with $f_n(x) = xf_{n-1}(x) + f_{n-2}(x)$ for $n \geq 2$, we obtain Fibonacci and Lucas *polynomials* and their generalizations. There are links to combinatorics, tilings, and graphs.

Kraft, James S. & Washington, Lawrence C., *An Introduction to Number Theory with Cryptography*, CRC Press, xvii + 554 pp., ISBN 9781482214413.

Well-written introductory text with cryptographic applications. It includes a number of topics not found in most introductory texts, such as the postage stamp problem, queens on a chessboard, polynomial congruences, Midy's theorem on decimal expansions, card shuffling, the discrete log problem, computing square roots mod p , volumes and Minkowski's theorem, and algebraic integers. There is a chapter on analytic methods and a very short chapter on Fermat's Last Theorem.

Kraft, James S. & Washington, Lawrence C., *Elementary Number Theory*, CRC Press, 2015, hardcover, xvi + 393 pp., ISBN 97881498702683.

This is a more compact and accessible version of the preceding text. It leaves out the chapters on the geometry of numbers and algebraic numbers and shortens some of the discussions, but still includes more than enough material for an introductory course.

Kumanduri, Ramanujachary & Romero, Cristina, *Number Theory with Computer Applications*, Prentice Hall, 1998, hardcover, xiii + 543 pp., ISBN 013801812X.

Well-written text with numerous applications and computer experiments. Various number theoretic methods are written in algorithmic form to make them easier to program on a computer, and the authors make electronic resources available on their web site, <http://www.math.columbia.edu/~rama/book.html>. Includes projects and historical notes. Chapters on cryptography, primality testing and factoring, Diophantine approximations, distribution of primes, binary quadratic forms, and elliptic curves.

Landau, Edmund, *Elementary Number Theory*, Chelsea, 1958, hardcover, 256 pp.

The content of this book was originally published in German in 1927 as part of a three volume set entitled *Vorlesungen über Zahlentheorie*. Each volume had two sections. This book is the first section of the first volume, the section most accessible to students. It consists of four parts: Foundations of Number Theory; Brun's Theorem and Dirichlet's Theorem; Decomposition into Two, Three, and Four Squares; and The Class Number of Binary Quadratic Forms. Exercises have been added by Paul T. Bateman and Eugene E. Kohlbecker.

LeVeque, William J. (Judson), *Elementary Theory of Numbers*, Dover 1990 (corrected reprint of 1962 Addison-Wesley edition), paperback, viii + 132 pp., ISBN 0486663485.

This compact and accessible book has seven chapters entitled Introduction, The Euclidean Algorithm and Its Consequences, Congruences, The Powers of an Integer Modulo m , Continued Fractions, The Gaussian Integers, and Diophantine Equations. Answers to selected problems are included.

LeVeque, William J. (Judson), *Fundamentals of Number Theory*, Dover (reprint of 1977 Addison-Wesley edition), paperback, vii + 280 pp., ISBN 0486689069.

More advanced than Dudley and Burton, but less advanced than Grosswald or Hardy & Wright. Covers some algebraic and analytic number theory topics. Contains historical notes and references to the literature.

LeVeque, William J. (Judson), *Topics in Number Theory*, Dover, 2002, (reprint of two-volume 1956 Addison-Wesley edition), paperback, 493 pp. (Vol I, xii + 202 pp., Vol II, vi + 273 pp.), ISBN 0486425398.

Volume I is a suitable text for advanced undergraduates and beginning graduate students. Volume II requires a much higher level of mathematical maturity, with contents including binary quadratic forms, applications of algebraic numbers to rational number theory, the Thue-Siegel-Roth theorem, irrationality and transcendence, Dirichlet's theorem on primes in arithmetic progressions, and the Prime Number Theorem.

Long, Calvin T., *Elementary Introduction to Number Theory* (Third Edition), Waveland Press, 1995, hardcover, xii + 292 pp., ISBN 0881338362. Reprint of the 1987 Prentice Hall edition.

Good readable introductory text. Suitable for self-study. Emphasizes the Fibonacci sequence. Has chapters on cryptography and simple continued fractions.

Marshall, David C., Odell, Edward, & Starbird, Michael, *Number Theory Through Inquiry*, The Mathematical Association of America, 2007, ix + 140 pp., ISBN 9780883857519, ebook ISBN 9780883859834.

This is a book that emphasizes the dual goals of developing mathematical thinking skills and developing an understanding of number theory. The goals are to have students look at examples and formulate definitions and questions or conjectures, prove theorems using various strategies, and determine the correctness of mathematical argument independently without having to ask an authority. Each chapter contains definition, examples, exercises, questions, and statements of theorems. Students are asked to answer the questions, do the exercises, prove most of the theorems (with guidance), and thereby develop the subject themselves by following the path laid out in the book. Students learn to study examples, make conjectures, and try to prove them. In that way, they experience what it is like to *do* mathematics rather than just try to learn what was worked out by others.

Miller, Steven J. and Takloo-Bighash, Ramin, *An Invitation to Modern Number Theory*, Princeton University Press, 2006, hardcover, xx + 503 pp., ISBN 0691120609.

An introductory text for *very* advanced readers. (Quadratic reciprocity appears on page 23!) Concentrates on methods of analytic number theory and presents many open problems. There are five parts: 1. Basic Number Theory (mod p arithmetic, group theory, and cryptography, arithmetic functions, zeta and L -functions, solutions to Diophantine equations), 2. Continued Fractions and Approximations (basic properties of algebraic and transcendental numbers, and Roth's theorem on how well algebraic numbers can be approximated by rationals), 3. Probabilistic Methods and Equidistribution (hypothesis testing, distribution of digits of continued fractions, introduction to Fourier analysis, Central Limit Theorem, and Poisson Summation), 4. The Circle Method (Goldbach's conjecture, Germain primes), and 5. Random Matrix Theory and L -Functions.

Mollin, Richard A., *Fundamental Number Theory with Applications* (Second Edition), Chapman & Hall/CRC, 2008, hardcover, x + 369 pp., ISBN 1420066595.

Very different from the first edition, which contained much advanced material. This is a thorough introduction to number theory that previews the wide scope of the subject early on, weaving many topics into the early chapters and pursuing them in more detail later on. The author provides interesting

exercises, numerous biographical sketches, and coverage of topics such as Thue's theorem; partition theory; generating functions; random number generation; Legendre's theorem on the equation $ax^2 + by^2 + cz^2 = 0$; Bachet's equation $y^2 = x^3 + k$; numbers of primitive representations of integers as sums of two, three, and four squares; primality testing; cryptography; and factorization methods. Appendix A reviews concepts from set theory, linear algebra, and abstract algebra. There are short appendices on the ABC conjecture and the recently discovered polynomial-time algorithm for primality testing. Solutions to the odd-numbered exercises are included.

Mollin, Richard A., *Advanced Number Theory with Applications*, Chapman & Hall/CRC, 2010, hardcover, xiii + 466pp., ISBN 9781420083286.

This book is designed as a second course in number theory at the senior undergraduate or junior graduate level to follow a course in elementary methods based on a book like the author's *Fundamental Number Theory with Applications*. The ten chapters are entitled Algebraic Number Theory and Quadratic Fields; Ideals; Binary Quadratic Forms; Diophantine Approximation; Arithmetic Functions; Introduction to p -Adic Analysis; Dirichlet: Characters, Density, and Primes in Progression; Applications to Diophantine Equations; Elliptic Curves; and Modular Forms. There is an appendix on sieve methods. Solutions to the odd-numbered exercises are included.

Moreno, Carlos J. and Wagstaff, Samuel S., *Sums of Squares of Integers*, Chapman & Hall/CRC, 2006, hardcover, xi + 354 pp., hardcover ISBN 1584884568, ebook ISBN 9781420057232.

An advanced book that is an excellent source for some classic results in number theory. Gauss's theorem on sums of three squares is on pp. 42–43; Cauchy's proof of the polygonal number theorem (every number is the sum of three triangular numbers, four squares, five pentagonal numbers, etc.) is on pp. 54–57.

Nagell, Trygve, *Introduction to Number Theory*, Wiley, 1951, hardcover, 309 pp.

This is a high-level introduction to number theory. The chapter headings are Divisibility, On the Distribution of Primes, Theory of Congruences, Theory of Quadratic Residues, Arithmetical Properties of the Roots of Unity, Diophantine Equations of the Second Degree, Diophantine Equations of Higher Degree, and The Prime Number Theorem. The author warns that "Most of the exercises are not of a routine character but are really intended to supplement the theory with known and new results which are not otherwise included in the text."

Nathanson, Melvyn B., *Elementary Methods in Number Theory*, Springer, 2000, hardcover, xviii + 513 pp., ISBN 0387989129.

Despite its title, this is a high-level text. It is divided into three parts: A First Course in Number Theory, Divisors and Primes in Multiplicative Number Theory, and Three Problems in Additive Number Theory. Most of our course is contained in the first three of the 16 chapters.

Nelsen, Roger B., *Nuggets of Number Theory: A Visual Approach*, MAA Press (an imprint of the American Mathematical Society), 2018, ISBN 9781470443986.

This book provides visual approaches to several topics in number theory. It is not a textbook, but rather a resource designed to be used as a supplement to a standard textbook. Its chapter headings are Figurate Numbers, Congruence, Diophantine Equations, Pythagorean Triples, Irrational Numbers, Fibonacci and Lucas Numbers, and Perfect Numbers, but with the exception of the first one, the chapter headings don't hint at the profusion of illustrations that pervade the book. There are many examples, and solutions to all the exercises are included.

Niederreiter, Harald & Winterhof, Arne, *Applied Number Theory*, Springer, 2015, x + 442 pp., ISBN 9783319223209, ebook ISBN 9783319223216.

From the preface: "Today, number theory is implicitly present in everyday life: in supermarket barcode readers, in our cars' GPS systems, in the error-correcting codes at work in our smartphones, and in online banking, to mention but a few examples. From our perspective, there are four major areas of application where number theory plays a fundamental role, namely cryptography, coding theory, quasi-Monte Carlo methods, and pseudorandom number generation. Excellent textbooks are available for each of these areas. This book presents the first unified account of all these applications. This allows us to delineate the manifold links and interrelations between these areas." Indeed, there is a chapter for each of the four areas mentioned, but there is also a chapter on Further Applications, which includes check-digit systems, Hadamard matrices and their applications, number theory and quantum computation, Benford's law, and an application to raster graphics.

Niven, Ivan, Zuckerman, Herbert S., & Montgomery, Hugh L. *An Introduction to the Theory of Numbers* (Fifth Edition), Wiley, 1991, hardcover, xiii + 529 pp., ISBN 0471625469.

Elegant text with broad and deep coverage. Chapters on Diophantine equations and elliptic curves, Farey fractions, continued fractions, primes and multiplicative number theory, algebraic numbers, partitions, and density of integer sequences. Excellent problems.

Ore, Oystein, *Invitation to Number Theory*, Mathematical Association of America, 1967, paperback, viii + 129 pp., ISBN 0883856204.

Not a text. Gentle introduction to number theory suitable for high school students and others with minimal training in mathematics. Extremely readable. Touches on history, polygonal numbers, and magic squares. Covers Pythagorean triangles, numeration systems, and congruences, with applications to recreational problems, tournament schedules, and perpetual calendars.

Ore, Oystein, *Number Theory and Its History*, Dover, 1988, paperback, x + 370 pp., ISBN 0486656209. Reprint of 1948 McGraw-Hill book.

Elementary and readable text. Good on history. Discusses constructibility of regular polygons, extensions of Fermat's little theorem, and an application of number theory to the splicing of telephone cables. Does not cover quadratic reciprocity or polynomial congruences of degree higher than two. Not many exercises.

Pettoufrezzo, Anthony J. and Byrkit, Donald R., *Elements of Number Theory* (Second Edition), Orange Publishers, 1994, paperback, xii + 244 pp., ISBN 9996753786.

Introductory text with an emphasis on continued fractions. Doesn't cover quadratic reciprocity. Has sections on inverting Euler's phi function and on linear congruences in two variables.

Pommersheim, James E., Marks, Tim K., and Flapan, Erica L., *Number Theory: A Lively Introduction with Proofs, Applications, and Stories*, Wiley, 2010, hardcover, xxi + 763pp., ISBN 9780470424131.

This text is different from all others. It uses math myths (fictitious tongue-in-cheek tales about famous mathematicians) to illustrate fundamental concepts and link them to the people who used them to obtain important results in number theory. Other innovations include cartoon characters who motivate proofs, interpret results, or misuse them; interesting biographies. The authors succeed in making the text fun to read and include topics not easily found elsewhere, such as recognizing exaggerations of the importance and omnipresence of the golden ratio, applying least common multiples and greatest common divisors to bicycle gears and human pitch perception, and interpreting the Euclidean algorithm in terms of cutting a rectangular cake into square pieces.

Rademacher, Hans, *Lectures on Elementary Number Theory*, Krieger, 1977, hardcover, ISBN 0882754998.

This set of lectures is an unusual and fascinating introduction to number theory. Starting with Farey fractions, Rademacher proves the Fundamental Theorem of Arithmetic and goes on to discuss congruences, decimal fractions, the approximation of real numbers by rationals, primitive roots, the regular 17-gon, cyclotomic equations, Gaussian sums, quadratic reciprocity, lattices, the distribution of prime numbers, primes in arithmetic progression, and a theorem of Brun. The quality of the exposition is excellent, but as this is not a textbook, there are no exercises.

Redmond, Don, *Number Theory: An Introduction*, Marcel Dekker, 1996, hardcover, xii + 741 pp., ISBN 0824796969.

This long, but accessible, introductory text goes farther than usual, featuring detailed coverage of approximation of real numbers, Diophantine equations, arithmetic functions (including their average order), and prime number theory, plus an introduction to algebraic number theory.

Ribenboim, Paulo, *My Numbers, My Friends: Popular Lectures on Number Theory*, Springer, 2000, ix + 375 pp., ISBN 0387989110.

From the preface: "May these essays stimulate your curiosity and lead you to books and articles where these matters are discussed at a more technical level. I warn you, however, that the problems treated, in spite of being easy to state, are for the most part very difficult. Many are still unsolved. You will see how mathematicians have attacked these problems." The book discusses Fibonacci and Lucas numbers, representation of numbers by means of Fibonacci numbers, prime number records (some now out of date), production of primes, Euler's famous prime generating polynomial, consecutive powers, the reason the prime 1093 is special, powerful numbers, Fermat's Last Theorem, the equation $AX^\ell + BY^m = CZ^n$, the nature of $\sqrt{2}^{\sqrt{2}}$, continued fractions, transcendental numbers, special properties of some integers (11, 19, 29, 30, 32, 60, 61, 62, 64, and 65), and more. Many partial results and references are given.

Ribenboim, Paulo, *The New Book of Prime Number Records* (Third Edition), Springer-Verlag, 1996, hardcover, xxiv + 541 pp., ISBN 0387944575.

A compendium of facts about prime numbers and their distribution, complete with discussion, proofs, and extensive references.

Ribenboim, Paulo, *The Little Book of Bigger Primes* (Second Edition), Springer-Verlag, 2004, paperback, xxiii + 350 pp., ISBN 0387201696.

The author presents many of the fundamental results connected with prime numbers, with attention to elementary material, computational results, and easily understood proofs.

Riesel, Hans, *Prime Numbers and Computer Methods for Factorization* (Second Edition), Birkhäuser, 1994, hardcover, xvi + 464 pp., ISBN 0817632913.

This readable book focuses on computer primality tests and computer methods for factorization. Discusses the distribution of primes and applications to cryptography. Extensive appendices and tables. Some Pascal computer programs included. Does not discuss elliptic curves.

Robbins, Neville, *Beginning Number Theory* (Second Edition), Jones and Bartlett, 2006, hardcover, xi + 338 pp., ISBN 0763737682.

An introductory book on number theory featuring chapters on arithmetic functions, continued fractions, nonlinear Diophantine equations, computational number theory, and cryptology. Some of the material on Fibonacci and Lucas numbers and binary linear recurrences from the first edition has been deleted.

Roberts, Joe, *Elementary Number Theory: A Problem Oriented Approach*, MIT Press, 1977, paperback, vi + 646 pp., ISBN 0262680280.

This unusual book is literally written by hand, in calligraphy. It presents number theory through a large collection of problems, with all solutions included. Goes farther and deeper than most elementary books. Includes theorems of Kronecker, Beatty, Skolem, Brun, Lucas-Lehmer, Weyl, and Dirichlet. Some analytic number theory. (See also Burn's *A Pathway Into Number Theory*.)

Roberts, Joe, *Lure of the Integers*, Mathematical Association of America, 1992, paperback, xvii + 310 pp., ISBN 088385502X.

This book describes interesting properties of particular integers. For example, under 1729, we find the fact that 1729 is the smallest positive integer expressible as the sum of two cubes in two different ways ($10^3 + 9^3$ and $12^3 + 1^3$). The entries are arranged in the order of the integers considered. (See also *The Penguin Dictionary of Curious and Interesting Numbers* by Wells.)

Rose, H. E., *A Course in Number Theory*, Second Edition, Oxford University Press, paperback, 1995, xv + 398 pp., ISBN 0198523769.

This advanced text covers our core material in about 70 pages. After that, it covers topics like Gauss and Jacobi sums, transcendental numbers, quadratic forms, partitions, theorems on the distribution of primes, advanced theorems on Diophantine equations, and elliptic curves. It assumes a knowledge of basic linear algebra, some group theory and field theory, as well as calculus and some analysis. Answers are given for numerical exercises and hints or sketches of solutions are given for other problems.

Rosen, Kenneth H., *Elementary Number Theory and Its Applications* (Fifth Edition), Addison-Wesley, 2005, hardcover, xx + 721 pp., ISBN 0321237072.

Excellent coverage of modern applications of number theory. Good exercises; some are quite challenging. Problems include computer projects. Applications include computer operations with integers, factorization by the Pollard ρ method, Pollard's $p - 1$ method, perpetual calendars, round-robin tournaments, hashing functions, check digits, cryptology, splicing of telephone cables, zero-knowledge proofs, and factorization using continued fractions. This edition adds a chapter on Gaussian integers and an appendix on using Maple and Mathematica for number theory. Another appendix lists number theory web links. The book also has its own web site: <http://www.awlonline.com/rosen>.

Scharlau, Winfried and Opolka, Hans, *From Fermat to Minkowski—Lectures on the Theory of Numbers and Its Historical Development*, Springer, 1985, hardcover, xi + 184 pp., ISBN 0387909427.

This is not a text, but a course of lectures from 1977–78 aimed at future high school mathematics teachers in Germany. It is not meant as a systematic introduction to number theory but rather as a historically motivated invitation to the subject, designed to interest readers in number-theoretical questions and developments. It tries to show how the investigation of obvious or natural questions has led to more and more comprehensive and profound theories. It covers a mix of topics, some of them quite advanced.

Schroeder, M. R. (Manfred Robert), *Number Theory in Science and Communication: With Applications in Cryptography, Physics, Digital Information, Computing, and Self-Similarity* (Fifth Edition), Springer-Verlag, 2009, hardcover, xxiv + 432 pp., ISBN 9783540852971.

Not a text. Contains an amazing array of applications of number theory to subjects including pitch perception, acoustics, electrical networks, cryptography, Doppler radar, computing, self-similarity, etc.

Schumer, Peter D., *Introduction to Number Theory*, PWS Publishing Company, 1996, hardcover, xi + 287 pp., ISBN 0534946267.

An accessible introductory text, with lots of problems, some historical comments, and chapters on continued fractions and Farey sequences, factoring and primality testing, analytic number theory, and additive number theory.

Shanks, Daniel, *Solved and Unsolved Problems in Number Theory* (Fourth Edition), Chelsea, 1993, hardcover, xiii + 305 pp., ISBN 082182824X.

An interesting and unusual presentation of elementary number theory, organized into three long chapters (From Perfect Numbers to the Quadratic Reciprocity Law, The Underlying Structure, and Pythagoreanism and Its Many Consequences), plus an appendix and a report on recent progress.

Shapiro, Harold N., *Introduction to the Theory of Numbers*, Wiley-Interscience, 1983, hardcover, xii + 459 pp., ISBN 0471867373.

Despite its title, this is an advanced book. It covers the standard topics from a sophisticated point of view, digressing in unexpected directions. Two “do-it-yourself” chapters save space by leaving many proofs to the reader. Discusses the ring of arithmetic functions, structure of the abelian group of reduced residue classes, counting problems, primes in arithmetic progression, and the prime number theorem.

Shoup, Victor, *A Computational Introduction to Number Theory and Algebra*, Cambridge University Press, 2005, hardcover, 517 pp., ISBN 0521851548. Version 2, 2008, forthcoming in print. Both versions freely available electronically in pdf format at <http://www.shoup.net/ntb/>.

The author states that his goal was to provide an introduction to number theory and algebra, with an emphasis on algorithms and applications, that would be accessible to a broad audience. The main prerequisite is that readers should be comfortable with mathematical formalism and have some experience in reading and writing mathematical proofs. All mathematics required beyond basic calculus is developed “from scratch”. The text does cover many topics not ordinarily found in elementary texts, such as distribution of primes, finite and discrete probability distributions, probabilistic algorithms, finding generators and discrete logarithms in \mathbb{Z}_p^* , computing modular square roots and inverses, applications of Gaussian elimination, linearly generated sequences and their applications, and algorithms for finite fields.

Sierpinski, Waclaw, *Elementary Theory of Numbers*, Panstwowe Wydawnictwo Naukowe (Poland), 1964, 480 pp. Second English edition, North-Holland, 1988, xii + 513 pp., ISBN 0444866620.

Excellent comprehensive text with considerably more depth than most introductory books.

Silverman, Joseph H., *A Friendly Introduction to Number Theory* (Third Edition), Prentice Hall, 2001, hardcover, vii + 434 pp., ISBN 0131861379.

This informal introductory text tries to get its readers to discuss and discover results in number theory. There are not many exercises—just enough so that readers can try them all, according to the author. Includes material on cryptography, Diophantine approximation, continued fractions, Gaussian integers, irrational and transcendental numbers, and elliptic curves. Be sure to visit and bookmark the text’s web page, <http://www.math.brown.edu/~jhs/frint.html>.

Sominskii, I. S., *The Method of Mathematical Induction*, Blaisdell, 1961, paperback, vii + 57 pp.

This isn’t a number theory text, but a little book on mathematical induction. It has many examples of induction proofs and lots of solved problems. The language is sometimes a bit formal, probably the result of its translation from the original Russian. Comparable to the book by Youse.

Stark, Harold M., *An Introduction to Number Theory*, MIT Press, 1978, paperback, x + 347 pp., ISBN 0262690608.

Very well written. Chapters on magic squares; rational, irrational, and transcendental numbers; continued fractions from a geometric viewpoint; and quadratic fields. Excellent introductory chapter. Nothing on quadratic reciprocity.

Stein, William, *Elementary Number Theory: Primes, Congruences, and Secrets—A Computational Approach*, Springer, 2009, x + 166 pp., hardcover ISBN 9780387855240, ebook ISBN 9780387855257.

This is a book about prime numbers, congruences, secret messages, and elliptic curves. Throughout the text, there are examples of calculations done using Sage. The chapter headings are Prime Numbers, the

Ring of Integers Modulo n , Public-key Cryptography, Quadratic Reciprocity, Continued Fractions, and Elliptic Curves.

Stewart, B. M. (Bonnie Madison), *Theory of Numbers* (Second Edition), Macmillan, 1964, hardcover, xiv + 383 pp.

A wonderful introductory text. Manages to cover a great deal of material in considerable depth without ever becoming stuffy or formal. Topics include Red Cross Solitaire, systems of linear Diophantine equations, the theory of indices and a slide rule for problems mod 29, the inradius of Pythagorean triplets, homogeneous quadratic Diophantine equations, Egyptian fractions, quadratic forms, decimal representation of rational numbers, continued fractions, Peano's axioms for the natural numbers, and about 100 pages on number theory from the point of view of modern algebra. I'm prejudiced in favor of this book because it's the second edition of the book I used as a student. The first edition (1952) has a section on inversion of Euler's phi function.

Stillwell, John, *Elements of Number Theory*, Springer-Verlag, New York, 2003, hardcover, xii + 254 pp., ISBN 0387955879.

This is an inviting and modern algebraic treatment of elementary number theory, with chapters on the RSA cryptosystem, the Pell equation, Gaussian integers, quadratic integers, the four square theorem, rings, ideals, and prime ideals. Some topics, such as primitive roots, Pell's equation, and quadratic forms, are covered surprisingly early in the book, making them more accessible. The author introduces the recent visual theory of quadratic forms from John H. Conway's book *The Sensual (Quadratic) Form*, Mathematical Association of America, 1997, xiii + 152 pp.

Strayer, James K., *Elementary Number Theory*, Waveland Press, 1994, reissued 2000, hardcover, xii + 290 pp., ISBN 1577662245.

A solid introductory text with standard topical coverage, featuring historical notes and interesting end-of-chapter student projects. There is a chapter on continued fractions and a chapter of applications, including RSA encryption, primality testing, Pell's equation, and a game called Square-Off.

Takloo-Biglash, Ramin, *A Pythagorean Introduction to Number Theory: Right Triangles, Sums of Squares, and Arithmetic*, Springer, 2018, xviii + 279 pp., ISBN 9783030026035.

This book was motivated by the problem of finding the number of right triangles with integer sides whose hypotenuses are bounded by a large number X . In attempting to give an elementary explanation of the solution, the author ended up introducing quite a bit of number theory, including the Gauss circle problem, the Möbius function, partial summation, and more. More generally, his idea was to ask natural number theoretic questions about right triangles and develop the necessary theory to answer those questions. Throughout the book, there are many places where geometric, topological, and analytic considerations play a role, illustrating the interplay of different parts of mathematics with number theory. Part I is foundational material, similar to what is found in other elementary number theory books. Part II is advanced topics: counting Pythagorean triples modulo an integer, counting lattice points on a circle or a sphere, the geometry of numbers, quadratic forms, and the distribution of rational points on the real line and the unit circle. A brief introduction to SageMath is included in an appendix.

This book was chosen as one of the "500 essential textbooks" that Springer made freely available to help educators, students and academics affected by the coronavirus. You can download it from Springer's website.

Tattersall, James J., *Elementary Number Theory in Nine Chapters* (Second Edition), Cambridge University Press, 2005, paperback, xi + 430 pp., ISBN 0521615240.

Introductory text with an emphasis on historical perspective. Chapters on perfect and amicable numbers; congruences of higher degree; cryptology; partitions; and representations of numbers as sums of powers, as binary quadratic forms, as continued fractions, and p -adically. Generous selection of exercises. Answers to selected exercises included.

Uspensky, J. V. (James Victor) and Heaslet, M. A. (Maxwell Alfred), *Elementary Number Theory* (Second Edition), McGraw-Hill, 1939, hardcover, x + 484 pp.

A classic introductory text. Readable, covers all the standard topics and many nonstandard ones including magic squares, calendar problems, card shuffling, Bernoulli numbers, quadratic forms, and Liouville's methods.

Vanden Eynden, Charles, *Elementary Number Theory* (Second Edition), Waveland Press, 2001, (reissue of McGraw-Hill edition), hardcover, viii + 278 pp., ISBN 1577664451.

This volume aspires to involve students actively in the construction of proofs and develop their mathematical self-confidence. The author tries hard to motivate nonobvious ideas. Special topics include

primality testing, public-key cryptography, coin-flipping by telephone, continued fractions, decimal representation of fractions, sums of powers, and Pell's equation.

Venkov, B. A., *Elementary Number Theory*, Wolters-Noordhoff, 1970, v + 249 pp., ISBN 9001896200.

This is an advanced book. Topics are introduced in rapid succession and proofs are very brief. For example, the proof of quadratic reciprocity ends on page 18. The chapter headings are Basic concepts of the theory of numbers, Continued fractions and the diophantine approximations, Power residues, Gauss's theory of quadratic forms, Partitions: Liouville's methods, and The class number of binary quadratic forms.

Vinogradov, I. M., *Elements of Number Theory*, Dover, 1954, viii + 227 pp.

This is a short introductory text. The chapter headings are Divisibility Theory, Important Number-Theoretical Functions, Congruences, Congruences in One Unknown, Congruences of Second Degree, and Primitive Roots and Indices. That material takes up 132 pages. The remaining 95 pages give the solutions to the problems.

Wagstaff, Samuel S., Jr., *The Joy of Factoring*, American Mathematical Society, 2013, xiv + 293 pp., ISBN 9781470410483.

This book is about the theory and practice of integer factorization, presented in a historic perspective. It's written for readers who want to learn more about the best methods of factoring integers, the reasons why factoring is important and useful, and the history of factorization methods. It can be read by anyone who has taken a first course in number theory. Answers and hints for exercises are included.

Watkins, John J., *Number theory: A Historical Approach*, Princeton University Press, 2014, hardcover, xiv + 576 pp., ISBN 9780691159409.

An introductory text that attempts to show how the great contributors to the subject thought about it and how it developed around the problems they were trying to solve. Unusually interesting problems, most with hints and/or solutions in the back of the book. Presentation is much more expository than in typical texts.

Weil, André, *Number Theory—An approach through history from Hammurapi to Legendre*, Birkhäuser, 1984, hardcover, xxi + 375 pp., ISBN 0817631410.

This is an historical approach to number theory, with numerous references to original sources. The bulk of the book is a detailed study and exposition of the achievements of Fermat, Euler, Lagrange, and Legendre, the founders of number theory. Weil says that the greatness of Gauss lies in his having brought to completion what his predecessors had initiated and in inaugurating a new era in the history of the subject.

Weil, André, *Number Theory for Beginners*, Springer, 1979, paperback, 70 pp., ISBN 9780387903811.

An extremely concise introduction to number theory from the point of view of abstract algebra. No previous acquaintance with abstract algebra is assumed. The only prerequisite is mathematical maturity—the ability to absorb new mathematical concepts readily and think about them abstractly.

Weissman, Martin H., *An Illustrated Theory of Numbers*, American Mathematical Society, 2017, hardback, xvi + 323 pp., ISBN 9781470434939.

This large format, heavily illustrated book is unique among introductions to number theory. Its chapter headings are The Euclidean Algorithm, Prime Factorization, Rational and Constructible Numbers, Gaussian and Eisenstein Integers, The Modular Worlds, Modular Dynamics, Assembling the Modular Worlds, Quadratic Residues, The Topograph, Definite Forms, and Indefinite Forms. The nearly 500 illustrations elucidate proofs, provide data visualization, and give fresh insights. The page layout is impressive. A new paragraph begins at the top of nearly every page. All theorems are proved and proofs fit on two facing pages, and usually on just one page. There are many examples solved, always on a single page. In the fourth chapter, facing pages treat Gaussian and Eisenstein integers on the left and right sides respectively. Footnotes appear beside their references in the wide right margins, not at the bottom. Each chapter begins with a figure on the left side and text on the right side of a two-page spread. Chapters end with historical notes and varied exercises, each exactly filling two facing pages.

Wells, David G., *The Penguin Dictionary of Curious and Interesting Numbers*, Penguin Books, 1986, paperback, 229 pp., ISBN 0140080295.

This book is similar in nature to Roberts' *Lure of the Integers*, with entries arranged in the order of the integers considered, but it contains different facts, including some for numbers that are not integers.

Wells, David G., *Prime Numbers: The Most Mysterious Figures in Math*, Wiley, 2005, xv + 272 pp., ISBN 0471462349.

A robust collection of facts, conjectures, and concepts involving prime numbers, arranged alphabetically with a glossary and a comprehensive index at the back. This is an enjoyable book to skim through, dip into at random, or read cover to cover. This is a popular book, not a work of scholarship, so references are given only for results that are not widely known. The material is arranged alphabetically by topic.

Youse, Bevan K., *Mathematical Induction*, Prentice-Hall, 1964, hardcover, 55 pp.

This isn't a number theory text, but a little book on mathematical induction. It has many examples of induction proofs and lots of solved problems. Comparable to the book by Sominskii, but written at a slightly lower level with less formal language. Theorem 2.4 is an unusual theorem on induction that I haven't seen elsewhere.

Zazkis, Rina & Campbell, Stephen R. (editors), *Number Theory in Mathematics Education: Perspectives and Prospects*, Lawrence Erlbaum Associates, 2006, x + 294 pp., ISBN 9780805854077.

The book consists of eleven essays on the potential of elementary number theory for mathematics education, such as "Understanding Elementary Number Theory in Relation to Arithmetic and Algebra", "On the Path to Number Theory: Repeating Patterns as a Gateway", "Learning by Teaching: The Case of Sieve of Eratosthenes and One Elementary School Teacher", "The Number-Theoretic Experience of 12- to 15-Year-Olds in a Calculator Environment: The Intertwining Coemergence of Technique and Theory", and "Learning Number Theory Concepts Via Geometrical Interactive Computerized Setting". It should appeal to mathematical educators, researchers in the field of mathematics education, and instructors focusing on the education of teachers.

OTHER LISTS. The Mathematical Association of America maintains a Basic Library List of books on various mathematical topics at <http://www.maa.org/BLL/TOC.htm>. The links at the bottom of the page lead to an explanation of the ratings, changes to the listings made in July 2010, and the subject index for the Basic Library List. For the number theory list, click on "The 1991 Basic Library List" and then on "Number Theory".

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